Caltech
Student-Faculty Programs

Session S Abstracts

Developing improved computational algorithms for the stable Andrews-Curtis conjecture
Elliot D. Steffen
Mentor: Sergei G. Gukov

This project investigates whether classical computer search and machine learning can make difficult problems in group theory more tractable. It focuses on the stable Andrews-Curtis conjecture, which asks whether symbolic descriptions of the trivial group can always be simplified by elementary moves. The project develops a stable move that temporarily introduces an additional symbol and performs a change of variables. The resulting algorithm also separates ordinary moves from this more expensive transformation, using it only when it is likely to improve the search. On a 60-case benchmark, the best setting solved every case, reducing the average number of search steps from 26,130 to 9,754 and the average running time from 552.71 seconds to 48.18 seconds. It did not solve additional difficult cases and usually produced longer solution sequences. These results show that selectively scheduling stronger transformations can reduce computational effort on difficult instances that were already solvable. Future work will explore machine learning integration to improve search decisions and solution finding.

Change of variables and difficulty measures for stable Andrews–Curtis search
Avigya Paudel
Mentors: Sergei G. Gukov, Lucas Fagan, and Michele Tarquini

The Andrews–Curtis conjecture, open since 1965, asks whether every balanced presentation of the trivial group can be simplified to a trivial presentation by a fixed set of moves; hard cases such as AK(3) have resisted every attempt. We study the conjecture's stable version via a change of variables (CoV) that stabilizes and destabilizes with arbitrary words (Lemma 11): a single "supermove" whose exact AC-move count is unknown, making it powerful enough to potentially solve previously unsolved presentations. On a benchmark of 66 presentations, the best transformed start solved 52 instead of 40; on the 40 both solved, it cut the search effort more than sevenfold and roughly halved the solution length. Automorphisms and AC moves reduce the 261 previously unsolved presentations to 124 equivalence classes; however, none were solved even with exhaustive CoV search. A random transform performs no better on average than direct search; the gain comes from generating many starts and keeping the best. A simple block-repetition measure separates solved from unsolved cases with 94 percent accuracy; a related tangling measure remains low for solved cases; and steering the search with these measures raises solves from 29 to 38 of 60. This offers an opportunity for machine learning: selecting the right change of variables at the right presentation state.

Computing F_K invariants of Montesinos knots via quantum 3j- and 6j-symbols
Weibing Guo
Mentor: Sergei G. Gukov

The F_K invariant is a quantum knot invariant that admits a state-sum description using representations of the quantum group $U_q(sl_2)$. This project aims to develop an explicit method for computing the F_K invariant of Montesinos knots by deriving closed-form formulas for the matrices associated with arbitrary rational tangles. Our approach builds on the quantum 3j- and 6j-symbols introduced by Solovyev and extends the construction beyond the highest weight setting to include mixed highest and lowest weight Verma modules. This requires rewriting the normalized quantum 3j- and 6j-symbols into a more convenient convention, establishing the corresponding graphical identities, and extending the formulas to the negative regime. The resulting framework provides the algebraic foundation for computing F_K invariants of Montesinos knots.

Positroid stratifications of positive symplectic Grassmannians
Henry Gustafson
Mentor: Yassine El Maazouz

Postnikov's positroid decomposition gives a combinatorial cell structure on the totally nonnegative Grassmannian. We study positive symplectic Grassmannians, consisting of nonnegative subspaces that are isotropic for a fixed symplectic form, by intersecting positroid cells with the symplectic equations. For two-planes in six-dimensional space, we classify the nonempty intersections and find six covering relations where the inherited positroid stratification fails. We also study explicit subtraction-free parametrizations of the resulting strata, investigate whether modified plabic graphs or network models can impose the symplectic constraints automatically, and compare the different positive geometries produced by different choices of symplectic form. We also explore extensions from Grassmannians to isotropic partial flag varieties using both positroid coordinates and Marsh–Rietsch parametrizations.

Tracial joint spectral measure for nearly commuting matrices
Chenning Xu
Mentor: Otte Heinaevaara

The tracial joint spectral measure is a measure over R^2 and originally introduced by Heinävaara to prove Hanner's inequality. We are interested in the measure itself: its continuous part, μ_c. μ_c is defined using two Hermitian matrices A and B, and associated with eigenvalues, although not the eigenvalues of A and B. We already know μ_c is compactly supported beforehand, and when A and B commute, it is zero everywhere. We perturb this trivial case a little bit. We let A and B start from diagonal matrices and apply an $O(\varepsilon)$ perturbation; then study the location, shape, and density of μ_c. For the location, we proved μ_c is supported near several collision lines, where two eigenvalues are equal. Near the collision lines, μ_c can be approximated by submatrices of A and B. The shape and error of approximation are tied to the size of the submatrices. 2×2 approximation is the simplest yet very crude: geometrically it gives a union of several ellipses and the error is at $O(\varepsilon)$. Larger sizes give reduced error but also have more complicated shape. Our questions on the shape and density of μ_c are answered by the submatrix approximation.

Infinitely many primes of basic reduction for some abelian sixfolds
Enoch J. Guo
Mentor: Elena Mantovan

Let E be an elliptic curve defined over a number field with at least one real embedding. Elkies proved, by a Euclidstyle argument, that E has supersingular reduction at infinitely many primes. More recently, Li, Mantovan, Pries, and Tang extended this result to certain abelian fourfolds defined over the rationals. In this project, we adapt their methods to further generalize this result to certain genus 6 curves admitting an automorphism of order 7, that is, we prove the Jacobians of such curves have basic reduction at infinitely many primes. More precisely, suppose that Ct is a smooth projective curve with an affine equation of the form Ct: y7= x(x-1)(x-t) defined over a number field k. We show that if there exists an embedding σ of k such that $\sigma(J(t))<27/4$, where J(t) is the J-function, then the Jacobian of Ct has basic reduction at infinitely many primes. Note that our result again holds for curves defined over arbitrary real number fields (satisfying the embedding condition), in the spirit of Elkies's original proof.

Formal languages and aperiodic spin chains on the boundaries of hyperbolic tilings
Alyna X. Tang
Mentor: Matilde Marcolli

Hyperbolic tilings are regular tessellations of hyperbolic space whose boundary degrees of freedom live on an aperiodic structure known as a conformal quasicrystal. The boundary geometry can be represented by strings encoding the local arrangement of tiles. Previous work developed a categorical framework connecting context-free languages with spin chain sets. We apply this framework to context-free languages that describe boundary configurations in hyperbolic tilings. Within a fixed tiling, we construct morphisms that encode inflation and regional growth, relating boundary languages

associated with different initial regions. We also construct morphisms between boundary languages arising from different hyperbolic tilings. We aim to use these constructions to investigate how transformations of the underlying geometry affect the physical properties of the associated spin chain sets.

Quantifying Gaussian decompositions of subgaussian random vectors and towards the convexity conjecture for solid sets
Caleb Pascale
Mentor: Antoine Song

We explicitly quantify the value of the uniform rescaling $\kappa \in \mathbb{R}$ present in the positive answer to the subgaussian vector problem, which states that for any centered 1-subgaussian random vector X in $\mathbb{R}^n$, we may write $\kappa X = G_1 + G_2 + G_3$, where $G_i \sim \mathcal{N}(0, I_n)$.

This result on sums of Gaussian vectors is equivalent to an affirmative answer to Talagrand's convexity problem. We then employ both geometric and probabilistic methods to obtain intermediate results towards a version of the conjecture for solid sets in $\mathbb{R}^n$.

The Schauder fixed point property for countable amenable groups
Lizzie V. Teryoshin
Mentor: Omer Tamuz

An amenable group G is said to have the Schauder fixed point property if every action of G on a compact convex set has a fixed point. For example, the infinite cyclic group has the Schauder fixed point property while the infinite dihedral group does not. Work of Oliver from 1975 classified which finite groups have the Schauder fixed point property, but the situation for infinite groups is still not known in general. In particular, it is unknown whether the lamplighter group $\mathbb{Z}_2 \wr \mathbb{Z}$ has the Schauder fixed point property. In joint work with Nicolas Monod, we show that $\mathbb{Z} \ltimes \mathbb{Z}_p$ (a quotient of the lamplighter group with lamps in $\mathbb{Z}_p$) and certain semidirect products by $\mathbb{Z}$, have the Schauder fixed point property.

Characterizing Andrews-Gordon summands in terms of subsets of cylindric partitions
Michelle Holland
Mentors: Tony Yue Yu, Weihong Xu, and Yifeng Huang

The Andrews-Gordon identities are Rogers-Ramanujan type identities, meaning they are *q*-series identities equating infinite sums with infinite products. In particular, they are direct generalizations of the classical two Rogers-Ramanujan identities, valued for their applications across many fields of math and physics. This project aims to enrich an existing proof of the Andrews - Gordon identities by giving an interpretation of each summand individually, as the original proof does not present a direct term-by-term correspondence. In that proof, the sum side is related as a whole to a generating function of cylindric partitions via B-paths, recovering the product side from a known product formula for this generating function. Our aim is to relate each summand individually to a particular subset of cylindric partitions, making this established connection more granular and structurally transparent. Time permitting, we also aim to eliminate the use of B-paths in the description of these subsets, which would allow for multi-row generalizations, and so would potentially lead to proofs of the conjectural HJO identities.