Caltech
Student-Faculty Programs

Session O Abstracts

Distributional exploration bonuses
Toby J. Saunders-A'Court
Mentors: Sergei G. Gukov and Muhammad A. Shehper

We introduce a new exploration bonus for reinforcement learning whose value is proportional to the information gain of a estimate distribution of returns updated with the distributional Bellman operators introduced in (Bellemare, 2016 / C51) and (Bellemare, 2018 / Quantile Regression). This method of combining exploration bonuses and distributional perspectives on reinforcement learning is predicted to generalize well other distributional methods commonly used in modeling partially observable Markov decision processes. We also propose a similar psuedo-count-based method for likewise combining these two aspects of reinforcement learning.

Multivariable matrix monotone functions
John G. Mattson, Jr.
Mentor: Otte Heinaevaara

Loewner's theorem characterizes single-variable functions that preserve the Loewner ordering (matrix monotone functions). Dobsch discovered the Dobsch matrices in 1937 and showed that positivity of the Dobsch matrices is a necessary condition for matrix monotonicity in one dimension. Donoghue later showed that this positivity is also sufficient. We study multivariable analogs of matrix monotone functions, as presented and defined in the paper of Agler, McCarthy, and Young (*Ann. Math*, 2013). In particular, we study the locality of multivariable matrix monotonicity, including analogs for the Dobsch matrices that characterize fixed-size matrix monotonicity in the single-variable case. We construct paths of tuples of matrices in order to extract potential Dobsch analogs in the multivariable case. Future work should involve finding a general form of Dobsch analogs for a general setting and potentially seeing if their positivity is sufficient for local matrix monotonicity in the sense of Agler, McCarthy, and Young.

Generalized tube algebras for locally presented quasi-local algebras
Jun Ikeda
Mentors: Corey Jones and Matilde Marcolli

Tube algebras associated with fusion spin chains play a central role in analyzing their categorical structure; in particular, their representation category is canonically equivalent to the Drinfeld center of the underlying fusion category. In this work, we extend that framework by constructing a generalized tube algebra for an abstract quasi-local *-algebra covariant under an isometric group action (for example, translation) on a countable metric space with bounded geometry, assuming local generation and local presentation. We then use this construction to define a fully faithful functor from the category of translation-invariant quasi-local *-algebras over the integers to the category of inverse systems of tube algebras of growing circumference modulo finite differences. Finally, in the case of fusion spin chains, we show that the tube algebra construction intertwines with the DHR functor and we develop a criterion for when a tensor autoequivalence of a DHR category cannot be realized by a quantum cellular automaton on the underlying quasi-local algebra.

Unitary Eichler basis theorems
Samuel P. Goodman
Mentor: Elena Mantovan

The goal of this project is to extend the Eichler basis problem on the level of superspecial points beyond the classical (GL_2) and Hilbert modular surface settings (as considered by Marc-Hubert Nicole, yielding a basis of classical modular forms at level \Gamma_0(p) from series attached to the points of the superspecial locus), specifically to unitary settings. Since unitary settings no longer admit positive definite inner product forms, the classical theta series constructions must be modified, which we do using the Kudla theta lift framework that has become essential in the Kudla Program. We reinterpret the superspecial basis problem, explaining a bridge between superspecial points and automorphic forms in terms of Rapoport-Zink uniformizations. We attach theta series to each superspecial point of the special fiber at p of the Shimura variety over an imaginary quadratic field attached to U(1,n) at Iwahori level and use these theta series to construct a map of modules over the Iwahori-Hecke algebra, which we claim is an isomorphism on the Steinberg parts.

Analyzing the asymptotic behavior of the integrality gap for the Goemans-Linial SDP relaxation of the Uniform Sparsest Cut
Grace H. To
Mentor: Seung-Yeon Ryoo

The Sparsest Cut Problem is a fundamental graph cut problem that aims to find a cut (a two-part partition) of a graph with uniform demands between pairs of vertices such that the edge capacities per demand cut is minimized. The Goemans-Linial SDP relaxation gives the best known approximation ratio $\IG$ for this problem, and its value directly corresponds to how well metric spaces embed into ℓ_1; in the special case of uniform demands, $\IGunif$ corresponds to average distortions of metric spaces into ℓ_1. It was recently proven that $\IG\asymp \sqrt{\log n}$. However, the behavior of the integrality gap for the uniform case $\IGunif$ is still not well understood. Kane and Meka computationally prove a better lower bound for $\IGunif$ of $\exp(c\sqrt{\log\log n}$ for some universal constant $c>0$, and we aim to find an approach using metric embeddings to prove an equivalent lower bound.

Algebraic models of Merge and phases for tensor networks
Henry Gustafson
Mentor: Matilde Marcolli

In Chomsky's Minimalist Program, *Merge* is a set-formation operation combining two syntactic objects. The hierarchical structure building process can be described in terms of *phases*, successive complete substructures. Previous research has attempted to relate the Merge operation to physical models such as tensor networks and renormalization. We show that these models, based on matrix representations, are not sensitive to Merge's non-associativity or phase structure. This project develops algebraic structures suitable for tensor networks while preserving the non-associativity and phase structure not captured by naïve matrix models. We show that there exists a faithful representation of syntactic objects in a Jordan operator algebra. By assigning lexical items to polynomial vector fields and recursively mapping rooted trees to differential operators, we can obtain explicit finite matrices for each tree. We then present a concrete tensor network implementation plan, so that a more accurate model of Merge can be developed.

Cutoff window for the Gibbs sampler process
Colin H. La
Mentor: Lingfu Zhang

Chatterjee, Diaconis, Sly, and Zhang previously proved the precise cutoff time for the repeated average process, which serves as a powerful model for social science and physical phenomena. The Gibbs sampler process generalizes the repeated average process by replacing the fixed average with a random weighted average. This work investigates whether the Gibbs sampler process exhibits cutoff phenomena in terms of total variation distance and establishes bounds for the cutoff window. In this

paper, we approximate the Gibbs sampler process with an analogous branching structure and leverage techniques from large deviations theory to approximate tail distributions in the asymptotic regime. Identifying a precise cutoff window lends to a sharper understanding of the convergence behavior of the Gibbs sampler process, which has key implications for models in economic theory and physical systems.

Upper bounds on the optimal pebbling number of graphs with high minimum degree
Piash Mondol
Mentor: Matthew M. Gherman

Given a distribution of pebbles on the vertices of a graph G, a pebbling move takes two pebbles from one vertex and puts one on a neighboring vertex. The optimal pebbling number is the least k such that some distribution of k pebbles permits reaching each vertex. It is conjectured that the optimal pebbling number of a graph on n vertices with minimum degree at least 3 is at most the ceiling of n/2. We design a pebbling technique that proves the conjecture for graphs with minimum degree at least 5 and improves the known upper bound on the optimal pebbling number of graphs with minimum degree 3 or 4. Further, we find upper bounds and some lower bounds on the optimal pebbling number of 3-regular circulant graphs and grid graphs.

The negligible ideals of some symmetric groups, and its application to the Krull dimension of the integral negligible quotient of any finite group
Vivian Loh
Mentor: Matthew M. Gherman

For a finite group G and a G-module M, an element $u \in H^d(G,M)$ is negligible over $\mathbb{Q}$ if for every field extension $L/\mathbb{Q}$ and every continuous group homomorphism from $\mathrm{Gal}(L_{\mathrm{sep}}/L)$ to G, u is in the kernel of the induced homomorphism $H^d(G,M) \to H^d(L,M)$. When a G-module M is a unital ring and has a trivial G-action, the negligible elements form an ideal of the cohomology ring $H^*(G,M)$, denoted $I(G,\mathbb{Z})$. The negligible quotient is $H^*(G,\mathbb{Z})/I(G,\mathbb{Z})$. We compute a strict lower bound on the Krull dimension of the negligible quotient of $H^*(G,\mathbb{Z})$ for even $|G|$, and find that when $|G|$ is the product of distinct odd primes, the Krull dimension is zero. Generalizing a result from Serre, we prove that twice any class is negligible in $H^*(S_n,M)$ for finitely generated M with a trivial G-action. We compute the mod-2 negligible ideal for $S_{3\leq n\leq 8}$, the mod-p^k negligible ideal for S_n when $p \leq n < 2p$, the integral negligible ideal for S_3, and the Krull dimension of the integral negligible quotient for S_4. Analyzing restrictions to and from the alternating group, we compute the mod-2 negligible ideals for $A_{3\leq n\leq 5}$, the radical of the mod-2 negligible ideals for $A_{6\leq n\leq 9}$, and the mod-p^k negligible ideals when $p \leq n < 2p$.