Caltech
Student-Faculty Programs

Session S Abstracts

Estimating stationary distributions of Hopf algebra Markov chains in Merge
David Skigin
Mentor: Matilde Marcolli

After Noam Chomsky proposed his Minimalist thesis on language, Professor Marcolli developed an extensive mathematical treatment of the subject, firmly showing how some of the core computational structures in syntax can be interpreted through algebraic structures: specifically, Marcolli's construction relies on Hopf algebras, which elegantly capture the way sentences are combined and recombined through the Merge operator. An outstanding question in this field is the statistical action of Merge, namely if it is possible to find syntactic structures more favored in stationary distributions. Although this problem remains unsolved, I present a few potential approaches I developed over the summer to tackle this.

Tropical Hodge Conjecture for abelian fourfolds
Nancy Chen
Mentors: Tony Yue Yu, Shaowu Zhang, and Thorgal G. Hinault

The Tropical Hodge Conjecture posits that every tropical Hodge class arises from a tropical cycle via the cycle-class map. This project investigates the tropical Hodge conjecture for abelian fourfolds, an open question in tropical geometry. I began by explicitly constructing the eigenwave map on tropical chains and computing its matrix representation. I then computed the intersection of the kernel of this map over the reals with the domain lattice. The Hodge classes lie in this intersection, and I found exactly three such classes, which is consistent with results found in literature. To examine their algebraicity, I studied the cycle-class map. Preliminary results suggest these classes may correspond to actual tropical cycles, although the verification remains ongoing. This work contributes to a deeper understanding of tropical Hodge theory and offers insights towards the classical Hodge Conjecture, one of the most mysterious problems in algebraic geometry.

Shift invariance in half-space last passage percolation and directed polymers
Yunhao Lou
Mentor: Lingfu Zhang

Last passage percolation is a probabilistic model in which independent random weights are assigned to the vertices of the two-dimensional integer lattice, and one studies the maximum sum of these values along any directed path between two given points. The model is integrable when the weights are geometric or exponential. It has been proved that in those cases the model admits a nontrivial shift invariance in the joint distribution of passage times. We prove a similar invariance in the corresponding half-space models, where the environment is symmetric across the diagonal, and extend this result to a related directed polymer model with inverse-gamma weights.

Computations of and upper bounds on the optimal t-pebbling of graphs with rate r
Jacobo de Juan Millon
Mentor: Matthew M. Gherman

Given a distribution of pebbles on the vertices of a graph G and some r>1, a fractional pebbling move with rate r removes rx pebbles from a vertex and adds x on a single neighbor, for some $x \in \mathbb{R}_+$. The fractional optimal pebbling number with rate r, denoted $f^*(G, r)$, is the minimum real number k such that some distribution of k pebbles permits reaching each vertex by at least one pebble via fractional pebbling moves. With r>1 an integer, a rate r pebbling move is a fractional pebbling move with rate r and x=1. The optimal t-pebbling number with rate r, denoted $\pi^*_t(G, r)$, is the minimum number k such that some distribution of k pebbles permits reaching each vertex by at least t pebbles via rate r pebbling moves. All prior work in this field fixed the rate at r=2. We give a sharp upper bound for $f^*(G,$

r) in terms of $|V(G)|$ and the graphs for which the equality holds. We compute every $\pi^{*}{}_{t}(P_n, r)$ value and certain $\pi^{*}{}_{t}(G, r)$ values for cycles and trees with degree at most r+1. To bound $\pi^{*}{}_{t}(G, r)$ in terms of $n=|V(G)|$, we provide values of t such that $\pi^{*}{}_{t}(G, r) \leq \pi^{*}{}_{t}(P_n, r)$ for any graph G.

Investigation of the generalisability of reinforcement learning algorithms on mathematical environments by a case study of the Andrews-Curtis conjecture

Zhongyuan Li
Mentors: Sergei G. Gukov and Muhammad A. Shehper

This study aims to investigate the generalisability of some important reinforcement learning(RL) algorithms in mathematical environments. In the study, we use the Andrews-Curtis conjecture as the maths environment for the case study. Standard baseline RL algorithms such as A2C is first implemented to obtain baseline performance on the generalisability in this environment. The algorithm of Phasical Policy Gradient is then implemented and the results are compared with the baselines.